ឧទាហរណ៍-គ

គេតាង $y = (x + \sqrt{1+x^2})^2$

គណនា y′, y′′. បង្ហាញថា $(1+x^2)y'' + xy' - 4y = 0$ ។

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យើងតាង u = $\sqrt{\mathbf{1} + \boldsymbol{x}^{\mathbf{2}}}$ **=>** $\boldsymbol{y} = (\boldsymbol{x} + \boldsymbol{u})^{\mathbf{2}}$ **=>** $y' = 2(x+u)(1+u')$ **(1)**

$u = (1+x^2)^{\frac{1}{2}} \Rightarrow u' = \frac{1}{2}(1+x^2)^{\frac{1}{2}-1}(2x) = x(1+x^2)^{-\frac{1}{2}}$ **(2)**

$y' = 2(x + xu' + u + uu')$ (ដោយ (1))

$y' = 2[x + x.x(1+x^2)^{-\frac{1}{2}} + (1+x^2)^{\frac{1}{2}} + (1+x^2)^{\frac{1}{2}}.x(1+x^2)^{-\frac{1}{2}}]$ (ដោយ **(2)**)

$y' = 2[x + x^2(1+x^2)^{-\frac{1}{2}} + (1+x^2)^{\frac{1}{2}} + x]$

$\boldsymbol{y' = 4x + 2x^2(1+x^2)^{-\frac{1}{2}} + 2(1+x^2)^{\frac{1}{2}}}$ **(3)**

ដូច្នេះ

$y'' = 4 + 4x.(1+x^2)^{-\frac{1}{2}} + 2x^2(-\frac{1}{2})(1+x^2)^{-\frac{3}{2}}(2x) + 2(\frac{1}{2})(1+x^2)^{-\frac{1}{2}}(2x)$

$y'' = 4 + 4x.(1+x^2)^{-\frac{1}{2}} - 2x^3(1+x^2)^{-\frac{3}{2}} + 2x(1+x^2)^{-\frac{1}{2}}$

$\boldsymbol{y'' = 4 + 6x(1+x^2)^{-\frac{1}{2}} - 2x^3(1+x^2)^{-\frac{3}{2}}}$ **(4)**

យើងតាង A = $(1+x^2)y'' + xy' - 4y$

ដូច្នេះ ដោយ (3), (4) ៖

A = $(1+x^2)[4 + 6x(1+x^2)^{-\frac{1}{2}} - 2x^3(1+x^2)^{-\frac{3}{2}}] + x[4x + 2x^2(1+x^2)^{-\frac{1}{2}} +$

$2(1+x^2)^{\frac{1}{2}}] - 4(x + \sqrt{1+x^2})^2$

A = $4(1+x^2) + 6x(1+x^2)^{\frac{1}{2}} - 2x^3(1+x^2)^{-\frac{1}{2}} + 4x^2 + 2x^3(1+x^2)^{-\frac{1}{2}} +$

$2x(1+x^2)^{\frac{1}{2}} - 4[x^2 + 2x\sqrt{1+x^2} + (1+x^2)]$

A = $8x(1+x^2)^{\frac{1}{2}} + 4x^2 - 4[x^2 + 2x\sqrt{1+x^2}] = 0$

ដូច្នេះ $(\mathbf{1} + \mathbf{x}^{\mathbf{2}})\boldsymbol{y}'' + \boldsymbol{xy}' - \mathbf{4}\boldsymbol{y} = \mathbf{0}$ ។

សង្កេត

$(1 + x^2)y'' + xy' - 4y = 0$ ជាសមីការឌីផេរ៉ង់ស្យែល ហើយ

$y = (x + \sqrt{1 + x^2})^2$ ជាចំលើយនៃសមីការឌីផេរ៉ង់ស្យែលនោះ ។