

5.2 គណនា $a_n (n \geq 1)$

ដោយគុណ (3) នឹង $\cos p\omega t (p \geq 1)$, គេបាន :

$$e(t) \cos p\omega t = a_0 \cdot \cos p\omega t + \sum_{n \geq 1} (a_n \cos n\omega t \cdot \cos p\omega t + b_n \sin n\omega t \cdot \cos p\omega t)$$

ដោយធ្វើអាំងតេក្រាលពី 0 ទៅ T, គេបាន :

$$\int_0^T e(t) \cos p\omega t dt = a_0 \int_0^T \cos p\omega t dt + \sum_{n \geq 1} \left[a_n \int_0^T \cos n\omega t \cdot \cos p\omega t dt + b_n \int_0^T \sin n\omega t \cdot \cos p\omega t dt \right]$$

ដោយ $\int_0^T \cos p\omega t dt = 0$ (មើលចំណាំ) , នោះយើងអាចសរសេរ

$$\int_0^T e(t) \cos p\omega t dt = \sum_{n \geq 1} (a_n I_n + b_n J_n) \quad \text{ដោយ}$$

$$I_n = \int_0^T \cos n\omega t \cdot \cos p\omega t dt \quad \text{និង} \quad J_n = \int_0^T \sin n\omega t \cdot \cos p\omega t dt$$

(4)

ដើម្បីគណនា I_n រឺ J_n យើងប្រើរូបមន្ត ៩៣ ហើយយើងបាន :

$$I_n = \int_0^T \frac{1}{2} (\cos(n\omega t + p\omega t) + \cos(n\omega t - p\omega t)) dt$$

$$= \frac{1}{2} \left[\int_0^T \cos(n+p)\omega t dt + \int_0^T \cos(n-p)\omega t dt \right]$$

ចំពោះ $n \neq p$ យើងបានដឹងរួចមកហើយថា (មើលចំណាំ)

$$\int_0^T \cos(n+p)\omega t dt = 0 \quad \text{និង} \quad \int_0^T \cos(n-p)\omega t dt = 0$$

ចំពោះ $n = p$

$$\int_0^T \cos(n+p)\omega t dt = \int_0^T \cos 2p\omega t dt = 0$$

$$\int_0^T \cos(n-p)\omega t dt = \int_0^T dt = |t|_0^T = T \quad (\text{ព្រោះ } \cos 0 = 1)$$

ដូច្នោះ

$$I_n = \begin{cases} 0 & \text{if } n \neq p \\ \frac{1}{2}T & \text{if } n = p \Rightarrow I_p = \frac{T}{2} \end{cases} \quad (5)$$

$$\begin{aligned} J_n &= \int_0^T \sin n\omega t \cdot \cos p\omega t \, dt \\ &= \int_0^T \frac{1}{2} [\sin(n\omega t + p\omega t) + \sin(n\omega t - p\omega t)] \, dt \quad (\text{ដោយរូបមន្ត §3}), \quad \text{រឺ} \\ J_n &= \frac{1}{2} \left[\int_0^T \sin(n+p)\omega t \, dt + \int_0^T \sin(n-p)\omega t \, dt \right] \\ J_n &= 0, \quad \text{ព្រោះ } \int_0^T \sin(n+p)\omega t \, dt = 0 \quad (\text{ដោយចំណាំ}) \\ \text{និង } \int_0^T \sin(n-p)\omega t \, dt &= 0 \quad (\text{if } p = n, \sin(n-p) = \sin 0 = 0) \end{aligned}$$

ដោយសង្ខេប, (4) ទៅជា :

$$\begin{aligned} \int_0^T e(t) \cos p\omega t \, dt &= \sum_{n \geq 1} a_n I_n + \sum_{n \geq 1} b_n J_n \\ &= a_p I_p + \sum_{\substack{n \geq 1 \\ n \neq p}} a_n I_n \quad (\text{ព្រោះ } J_n = 0) \\ &= a_p \frac{T}{2} \quad (\text{ដោយ (5)}) \end{aligned}$$

$$\text{ដូច្នោះ } a_p = \frac{2}{T} \int_0^T e(t) \cos p\omega t \, dt \quad \forall p \geq 1$$